

Testing and selection of cosmological models with $(1+z)^6$ corrections

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Abstract

In the paper we check whether the contribution of $(-)(1+z)^6$ type in the Friedmann equation can be tested. We consider some astronomical tests to constrain the density parameters in such models. We describe different interpretations of such an additional term: geometric effects of Loop Quantum Cosmology, effects of braneworld cosmological models, non-standard cosmological models in metric-affine gravity, and models with spinning fluid. Kinematical (or geometrical) tests based on null geodesics are insufficient to separate individual matter components when they behave like perfect fluid and scale in the same way. Still, it is possible to measure their overall effect. We use recent measurements of the coordinate distances from the Fanaroff-Riley type IIb (FRIIb) radio galaxy (RG) data, supernovae type Ia (SNIa) data, baryon oscillation peak and cosmic microwave background radiation (CMBR) observations to obtain stronger bounds for the contribution of the type considered. We demonstrate that, while ρ^2 corrections are very small, they can be tested by astronomical observations – at least in principle. Bayesian criteria of model selection (the Bayesian factor, AIC, and BIC) are used to check if additional parameters are detectable in the present epoch. As it turns out, the Λ CDM model is favoured over the bouncing model driven by loop quantum effects. Or, in other words, the bounds obtained from cosmography are very weak, and from the point of view of the present data this model is indistinguishable from the Λ CDM one.

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I. INTRODUCTION

Modern astronomy has led us to the cosmological concordance model (CCM). The interpretation of observational data suggest that we are living in almost spatially flat, low density, accelerating universe filled with dust-like matter [1, 2, 3]. In other words, at first sight there is nothing more to the universe than the simple Λ CDM model. In this model, apart from the observed baryonic matter we have two additional components. One is in the form of non-relativistic and cold dust matter contributing one third of the total energy density, while the second one, having negative pressure violating the strong energy condition, is contributing about two thirds of the total energy density. This second term is interpreted as non-zero cosmological constant Λ . It could be called the minimal model in that it is the simplest to fit the data so well.

Despite that, it is obvious such model is far from reality, and much attention is paid to possible extensions of physics in that area [4, 5]. Especially the early universe, where general relativity needs closer association with quantum theories, is open to modelling. In order to preserve the agreement with late epoch observations, the modifications need to introduce effects negligible at present, and dominating for early times, that is to say, decreasing with the scale factor a , or increasing with density ρ – if one assumes an expanding universe. The Loop Quantum Gravity offers the possibility of the description of the discreteness of spacetime at the Planck scale. The classical Λ CDM should be emergent from the more fundamental quantum model. Singularities in classical general relativity should be replaced by the quantum description of the very beginning of the gravity. For example the Loop Quantum Gravity cosmological model predicts the bounce instead of the initial singularity. For full description of the universe evolution both the quantum and classical regimes should be taken into account.

Here, we investigate the observational constraints on a contribution of ρ^2 type to the Hubble parameter $H^2(z)$, which could also follow from an equivalent term of type $(-)(1+z)^6$. Because we have no a priori information which physical theory is valid in the Universe at the Planck epoch all possibilities should be taken into account. The viable interpretations of the presence of this term in $H(a)$ come from the loop quantum, braneworld and non-Riemannian cosmologies.

A. Universe in Loop Quantum cosmology

One of the important, unsolved problem of the modern cosmology is the problem of initial conditions for the Universe. Because, at least from our point of view, the Universe is given in one

copy the initial conditions cannot be taken from “outside”.

Many cosmologists argue that problem of initial conditions for the Universe can be shifted to the Planckian epoch in which quantum effects were crucial. In this context, the idea that present expansion of the Universe was preceded by a contracting phase seems to be very attractive. Geometric effects in Loop Quantum Cosmology (LQC) predict the presence of ρ^2 modification to the Friedmann equation $H^2(a)$ of negative sign [6, 7] which modifies the early evolution leading to a bounce in the generic case.

The basic formula $H(z)$ which is used in cosmography assumes very special form in the case of loop quantum cosmology if effects of curvature are included. In this case effects of curvature cannot be simply included in a model by adding a noninteracting curvature fluid with energy density $\rho_k = -\frac{3k}{a^2}$, $p_k = -\frac{\rho_k}{3}$ because loops quantum effects manifest themselves in the form of a modification to the classical Friedmann equation. For example if we consider a closed model the explicit form is given by [7, 8, 9, 10]

$$H^2 = \left(\frac{\rho}{3} + \frac{\Lambda}{3} - \frac{1}{a^2} \right) \left(1 - \frac{\rho}{\rho_{\text{crit}}} - \frac{\Lambda}{\rho_{\text{crit}}} + \frac{3}{a^2 \rho_{\text{crit}}} \right) \quad (1)$$

where ρ_{crit} is related with Planck energy density $\rho_{\text{crit}} = \frac{\sqrt{3}}{16\pi^2 \gamma^2} \rho_{\text{Pl}}$, $\rho_{\text{Pl}} = \frac{2\pi}{hG^2}$ is Planck density and $\gamma \simeq 0.2375$ is the Immirzi parameter. [11]. With $\rho_{\text{crit}} \simeq 0.82 \rho_{\text{pl}}$ [12] and $\rho_{\text{pl}} = 5.155 \times 10^{96} \frac{\text{kg}}{\text{m}^3}$ it leads to $\rho_{\text{crit}} \simeq 4.19 \times 10^{93} \frac{\text{g}}{\text{cm}^3}$. Please note that only the first term in parentheses is corresponding right hand side of the Friedmann equation. Therefore, loop high-energy modifications of general relativity appear in the second term in parentheses. The classical general relativity limit can be obtained if ρ_{crit} goes to infinity and the second term approaches unity.

The effective Friedmann equation assumes the form

$$H^2 = \frac{\rho}{3} \left(1 - \frac{\rho}{\rho_{\text{crit}}} \right) + \text{other terms} \quad (2)$$

The applications of LQG methods to cosmology [13, 14, 15] offer the possibility of investigating the consequences of discreteness of spacetime on the quantum level. One can distinguish a quantum bounce from the classical one, for which $\rho \ll \rho_{\text{crit}}$ and the quantum corrections are negligible.

In the next point we pointed out similar ρ^2 modification of the Friedmann equation as in LQC (for the comparison of brane world scenarios and LQG cosmology see [11, 16, 17, 18, 19]).

Recently Bojowald pointed out the possibility of testing loops quantum effects using more precise data in the near future [20]. He pointed out that while quantum corrections from gravity are commonly expected to be small, a quantum structure of spacetime can produce potentially observable effects. Martin Bojowald argued that cosmological models of loops can be in principle

testable but there is no concrete proposal in this context. We indicate some possibilities of such testing. However, we must remember that the expansion history $H(z)$ measures only the kinematic variables [21]. We express some scepticism about the possibility of testing LQC effects by using such a cosmography. In our opinion, testing the large scale structure (extragalactic matter distribution) of the Universe would be more valuable.

B. Braneworld cosmological models

The braneworld cosmology is based on the idea that the gravitational field propagates in bulk spacetime while our observable universe is only a surface (called brane) embedded in the bigger space. This idea leads to the presence of an additional term in the Friedmann equation on the brane. (For review of cosmology with extra dimensions see [22, 23, 24, 25].)

In the present paper we consider a version of higher-dimensional cosmology in the Randall and Sundrum framework [26, 27], which was already tested to some degree with SN Ia observations [28, 29].

In this cosmology, the Friedmann equation assumes the following form [30, 31]

$$H^2 = \frac{\Lambda_4}{3} - \frac{k}{a^2} + \frac{8\pi}{3M_p^2}\rho + \epsilon \left(\frac{4\pi}{3M_5^3} \right)^2 \rho^2 + \frac{C}{a^4} \quad (3)$$

where Λ_4 is the 4 dimensional cosmological constant, $\epsilon = \pm 1$ and C is an integration constant whose magnitude as well as sign can depend on the initial conditions. In the present paper we take into account the third term of type ρ^2 . This term arises from the imposition of a junction condition for the scale factor of the brane. This condition has simple interpretation - physical matter fields are confined to the brane. Both negative and positive ϵ are possible mathematically because ϵ corresponds to the metric signature of the extra dimension [31].

Till now even the sign of ϵ remains an open question. Sahni and Shtanov [32] discussed some consequences of a special choice of $\epsilon = -1$. One should note that possibility that the parameter has negative sign is crucial because in this case models with timelike extra dimension can avoid initial singularity by bounce [33]. The presence of (negative) ρ^2 term in the right hand side of (3) leads to a contracting universe to bounce instead of big-bang curvature singularity: $\rho \rightarrow \infty$, $R_{abcd}R^{abcd} \rightarrow \infty$. If the bounce takes place then we have constraint on the value of brane tension σ : $|\sigma| \geq 1\text{MeV}$ because the bounce takes place at densities greater than during the nucleosynthesis. Note that from observational point of view the modification of type $(-\rho^2)$ were also recently investigated by authors in the context of two-brane model [34] and this correction also arises in classically

constrained gravity [35, 36].

The fifth term in (3) (called dark radiation) we put here equal to zero. Note that this term would decay as rapidly as a^{-4} and would not modify the early evolution as significantly as the third term. However, please note that non zero dark radiation term can have non-negligible impact on nucleosynthesis [37, 38].

C. Non-Riemannian cosmological models

The cosmological models basing on the so called non-Riemannian extension of general relativity theory have been studied for long time (for a review see [4, 39, 40]). Trautman introduced the first non-Riemannian cosmological models based on Einstein-Cartan theory [41]. They are a modification of general relativity theory by adding the torsion of the spacetime. In such models, a consequence of spin and torsion is adding to the Friedmann equation an additional term $\rho^2 \propto a^{-6}$. The main advantage of such a model is that the problem of initial singularity can be avoided due to spin effects. Among the different extensions of General Relativity, the so called metric-affine gravity (MAG) is recently being studied. In contrast to Riemann-Cartan theory, the connection is not longer metric which implies that covariant derivative of the metric does not vanish.

Recently some authors [42, 43, 44] have used the magnitude-redshift relation and observations of distant supernovae type SNIa to constrain the model parameters. In the case considered the modified Friedmann equation assumes the following form

$$H^2 = \frac{\rho}{3} - \frac{k}{a^2} - \frac{\Lambda}{3} + v \frac{\psi^2}{a^6}, \quad (4)$$

where the new constant v can be both negative or positive. The above equation can be rewritten to a new form:

$$\Omega_{k,0} + \Omega_{m,0} + \Omega_{\Lambda,0} + \Omega_{\psi,0} = 1, \quad (5)$$

where $\Omega_{\psi,0} \equiv v \frac{\psi^2}{H^2}$, $\Omega_{\psi} = \Omega_{\psi,0} a^{-6}$ and $\psi = \text{const}$ is the integration constant. Therefore non-Riemannian quantities in this model (torsion and non metricity) will modify very early stages of evolution and are negligible at late times. The additional density parameter $\Omega_{\psi,0}$ is related to the non-Riemannian structure.

Different astronomical observations from Fanaroff-Riley Type IIb radio galaxies (FRIIb RG), X ray gas mass fraction to SNIa data can be used to constrain the present value of the parameter Ω_{ψ} ($\Omega_{\psi}(z=0) = \Omega_{\psi,0}$). This gives limits to the possible non-Riemannian structure of the spacetime.

The above are main possible interpretations of the presence of a ρ^2 contribution in the Friedmann equation. We must remember that cosmography which bases on the behaviour of null geodesics maps the geometry and kinematics of the Universe in terms of $H(z)$ without reference to the particular structure of each contribution. Because it measures only average properties of matter density it is possible to come up with different forms of contribution leading to the same form $(-)(1+z)^6$ [45]. It was also showed in [45] that in the generic case, alpha varying models lead to a bouncing universe. Interestingly, similar a^{-6} modifications have also be obtained in the universes with varying constants [46].

All the models under consideration can be represented in terms of density parameters Ω_i . For example for the brane models we can use a set of parameters

$$\Omega_k = -\frac{k}{a^2 H^2} = -\frac{k}{\dot{a}^2}, \quad \Omega_m = \frac{\rho}{3H^2}, \quad \Omega_\Lambda = \frac{\Lambda_4}{3H^2}, \quad \Omega_{dr} = \frac{a^4}{3H^2}, \quad \Omega_{mod} = -\frac{\rho^2}{3H^2 \rho_{cr}} \quad (6)$$

where σ is the brane tension. Then equation (3) can be rewritten to the form [47]

$$\sum \Omega_i = 1 \quad (7)$$

Note that Ω_m and Ω_{mod} are not independent, i.e. $\Omega_{mod,0} = -\frac{\Omega_{m,0}^2}{\Omega_{loops,0}}$, $\Omega_{loops,0} = -\frac{\rho_{cr}}{3H_0^2}$. Moreover $\Omega_{loops,0}$ is fixed from theory if H_0 is known. With $H_0 = 65 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $\Omega_{m,0} \simeq 0.3$ it gives $\Omega_{loops,0} \simeq 5.24 \times 10^{122}$ which gives a contribution to $\Omega_{mod,0} \simeq 1.72 \times 10^{-124}$ only. It would be worth mentioning that there are other sources of correction which act as stiff matter for example self-gravitational corrections of an Ads black hole [48].

In our paper we consider standard matter with energy density ρ and pressure $p = w\rho$, $w = \text{const}$ rather than matter in the form of non-minimally coupled to gravity scalar field. Bojowald advocated the latter approach where matter emerges from the scalar field [49]. We assume that standard matter satisfies the conservation condition which will determine the dependence of energy density on scale factor (or redshift). This means that we are not considering the inverse volume effects that would modify the scalar field energy and then conservation conditions. Good news for our estimation is that for the case of closed model inverse volume effects are negligible if universe is on macroscopic level [10].

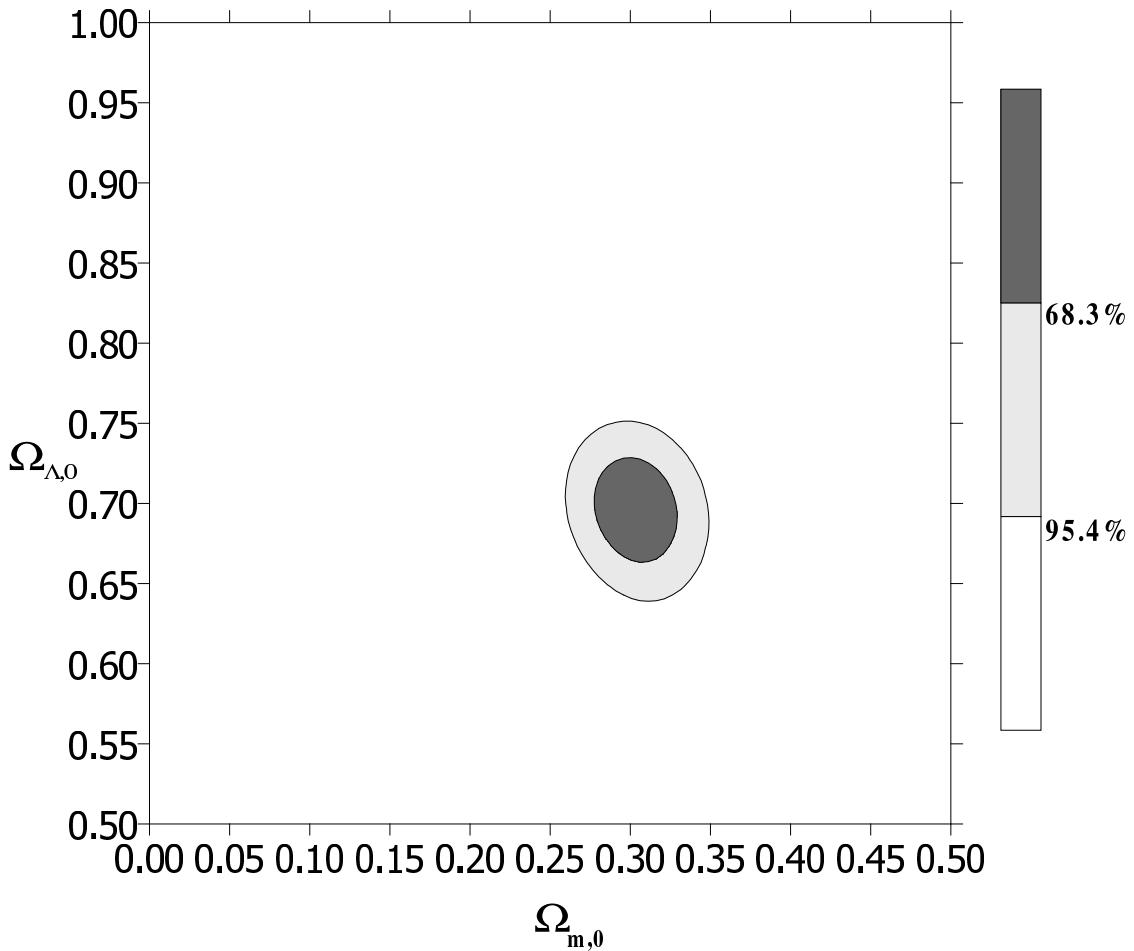


FIG. 1: The 68.3% and 95.4% confidence levels (obtained from combined analysis of SN+RG+SDSS+CMBR) on the $(\Omega_{m,0}, \Omega_{\Lambda,0})$ plane.

II. OBSERVATIONAL CONSTRAINTS ON THE FRW BOUNCING MODEL'S PARAMETERS

In comparison of the model with observational data we consider two different strategies. First of all, because the origin of the ρ^2 term is not a priori known we fitted a general $\Omega_{\text{mod},0}$ parameter from the observations. Accordingly, we obtained that it is too small for detection (probing) by astronomical observations (cosmography). In the second approach, we took into account the fact that in the LQG, the density parameter $\Omega_{\text{mod},0}$ is fixed. We consequently obtained that the model considered is indistinguishable from the ΛCDM model. It is a consequence of the theory itself determining the additional parameter's value to be very small.

Cosmological models are frequently tested against supernovae observations using the luminosity distance d_L of the Ia supernovae as a function of redshift [1]. With these types of tests for distant

SNIa, we can directly observe not the luminosity distance d_L but their apparent magnitude m and redshift z . Taking into account the fact that absolute magnitude \mathcal{M} of the supernovae is related to its absolute luminosity L , we can obtain the following relation between the distance modulus μ , the luminosity distance, the observed magnitude m and the absolute magnitude M

$$\mu \equiv m - M = 5 \log_{10} d_L + 25 = 5 \log_{10} D_L + \mathcal{M} \quad (8)$$

where $D_L = H_0 d_L$ and $\mathcal{M} = -5 \log_{10} H_0 + 25$. The luminosity distance of a supernova is a function of redshift and can be computed from the formulae:

$$d_L(z) = (1+z) \frac{c}{H_0} \frac{1}{\sqrt{|\Omega_{k,0}|}} \mathcal{F} \left(H_0 \sqrt{|\Omega_{k,0}|} \int_0^z \frac{dz'}{H(z')} \right) \quad (9)$$

where

$$\left(\frac{H}{H_0} \right)^2 = \Omega_{m,0}(1+z)^3 + \Omega_{k,0}(1+z)^2 + \Omega_{\text{mod},0}(1+z)^6 + \Omega_{\Lambda,0}, \quad (10)$$

$\Omega_{k,0} = -\frac{k}{H_0^2}$ and $\mathcal{F}(x) \equiv (\sinh(x), x, \sin(x))$ for $k < 0$, $k = 0$, $k > 0$, respectively.

We discuss, for greater generality, non-flat models and keep in mind that for LQG only the results obtained with fixed, zero k apply. This happens because LQG models' simple correction of the ρ^2 type appear in spatially flat universes, and assume a more complicated form for $k = \pm 1$ [13, 50].

Daly and Djorgovski [51] (see also [43, 52, 53]) suggest including in the analysis not only supernovae but also radio galaxies. They pointed out that for tests based on radiogalaxies it is useful to apply not the luminosity distance d_L but the coordinate distance $y(z)$ [54]. The relation between the luminosity distance d_L and the coordinate distance $y(z)$ has the following form

$$y(z) = \frac{H_0 d_L(z)}{c(1+z)}. \quad (11)$$

Daly and Djorgovski [55] have compiled a sample comprising the data on $y(z)$ for 157 SNIa in the Riess et al. [56] Gold dataset and 20 FRIIb radio galaxies. In our data sets we also include 115 SNIa compiled by Astier et al. [57].

From the comparison of eq. (10) and (11) it is easy to see that the coordinate distance $y(z)$ does not depend on the value of H_0 . Unfortunately, we do not observe the coordinate distance $y(z)$ of SNIa directly. This distance must be computed from the luminosity distance (or the distance modulus μ). It is clear that for such a computation a knowledge of the value of H_0 is required. For both supernovae samples we choose the values of H_0 which were used in the original papers. We used the distance modulus presented in Ref. [56, 57] for the calculation of the coordinate distance.

For each sample we choose the values of H_0 appropriate to the data sets. For Riess et al.'s Gold sample we have $h = 0.646$ as the best fitted value and this value is used for calculation of the coordinate distance for SNIa belonging to this sample. In turn, the value $h = 0.70$ was assumed in the calculations of the coordinate distance for SNIa belonging to Astier et al.'s sample, because the distance moduli μ presented in Ref. [57, Tab. 8] were calculated with such an arbitrary value of $h = 0.70$.

The error of the coordinate distance can be computed as

$$\sigma^2(y_i) = \left(\frac{10^{\frac{\mu_i}{5}}}{c(1+z)10^5} \right)^2 \left(\sigma^2(H_0) + \left(\frac{H_0 \ln 10}{5} \right)^2 \sigma^2(\mu_i) \right) \quad (12)$$

where $\sigma_i(\mu_i)$ denotes the statistical error of the distance modulus determination (note that for Astier et al.'s sample the intrinsic dispersion was also included) and $\sigma(H_0) = 0.8$ km/s Mpc denotes the error in H_0 measurements.

With the use of coordinate distance $y(z)$ as our basic quantity it is easy to include in our analyse two additional constraints which do not depend on the value of H_0 either. These constraints are obtained from extragalactic analysis. First, we have the baryon oscillation peaks (BOP) detected in the Sloan Digital Sky Survey (SDSS) Luminosity Red Galaxies [58]. They found that value of A

$$A \equiv \frac{\sqrt{\Omega_{m,0}}}{E(z_1)^{\frac{1}{3}}} \left(\frac{1}{z_1 \sqrt{|\Omega_{k,0}|}} \mathcal{F} \left(\sqrt{|\Omega_{k,0}|} \int_0^{z_1} \frac{dz}{E(z)} \right) \right)^{\frac{2}{3}} \quad (13)$$

(where $E(z) \equiv H(z)/H_0$ and $z_1 = 0.35$) is equal to $A = 0.469 \pm 0.017$. The quoted uncertainty corresponds to one standard deviation, where a Gaussian probability distribution has been assumed.

The second constraint which we include in our analysis is the so called (CMBR) “shift parameter”

$$R \equiv \sqrt{\Omega_{m,0}} y(z_{lss}) = \sqrt{\frac{\Omega_{m,0}}{|\Omega_{k,0}|}} \mathcal{F} \left(\sqrt{|\Omega_{k,0}|} \int_0^{z_{lss}} \frac{dz}{E(z)} \right)$$

where $R_0 = 1.716 \pm 0.062$ [59].

In our combined analysis, we can obtain a best fit model by minimizing the pseudo- χ^2 merit function [60]

$$\begin{aligned} \chi^2 &= \chi^2_{\text{SN+RG}} + \chi^2_{\text{SDSS}} + \chi^2_{\text{CMBR}} = \\ &\sum_i \left(\frac{y_i^{\text{obs}} - y_i^{\text{th}}}{\sigma_i(y_i)} \right)^2 + \left(\frac{A^{\text{mod}} - 0.469}{0.017} \right)^2 + \left(\frac{R^{\text{mod}} - 1.716}{0.062} \right)^2, \end{aligned} \quad (14)$$

where A^{mod} and R^{mod} denote the values of A and R obtained for a particular set of the model parameter. For Astier et al.'s SNIa sample [57] an additional error in z measurements was taken into account. Here $\sigma_i(y_i)$ denotes the statistical error (including the error in z measurements) of the coordinate distance determination.

We can obtain constraints for the cosmological parameters by minimizing the following likelihood function $\mathcal{L} \propto \exp(-\chi^2/2)$. One should note that when we are interested in constraining a particular model parameter, the likelihood function marginalized over the remaining parameters of the model should be considered [60].

Our results are presented in Table I, Table II and Fig. 1. Table I refers to the minimum χ^2 method, whereas Table II shows the results from the marginalized likelihood analysis. In Table III, Table IV we repeated our analysis with the prior $\Omega_{\text{mod},0} \leq 0$.

From our combined analysis (SN+RG+SDSS+CMBR), we obtain as the best fit a flat (or nearly flat universe) with $\Omega_{\text{m},0} \simeq 0.3$, and $\Omega_{\Lambda,0} \simeq 0.7$. For the $(1+z)^6$ term we obtain the stringent bound $\Omega_{\text{mod},0} \in (-0.26 \times 10^{-9}, 0.31 \times 10^{-4})$ at the 95% confidence level. These results mean that the positive value of $\Omega_{\text{mod},0}$ is preferred ($\Omega_{\text{mod},0} > 0$), however small negative contribution of $(1+z)^6$ type is also available. Our results shows that in the present epoch contribution of the dark radiation, if it exists, is small and gives only small corrections to the Λ CDM model in the low redshift.

The above analysis was performed for any spatial curvature. However, please note that it holds for non-flat models only if the Friedmann equation does not change in any other way after introducing the ρ^2 term. Unfortunately, LQG models with non-zero curvature were obtained for different setup of matter components [13, 50]. This means that for LQG only the flat case consideration apply.

The general basic formula for $H(z)$ for universe with $\Omega_{k,0} \geq 0$ rewritten in terms of dimensionless density parameters $\Omega_i = \frac{\rho_i}{3H_0^2}$ is

$$\left(\frac{H}{H_0}\right)^2 = (\Omega_{\text{m},0}(1+z)^3 + \Omega_{k,0}(1+z)^2 + \Omega_{\Lambda,0}) \left(1 - \frac{\Omega_{\text{m},0}(1+z)^3}{\Omega_{\text{loops},0}} - 3\frac{\Omega_{k,0}(1+z)^2}{\Omega_{\text{loops},0}}\right), \quad (15)$$

(where $\Omega_{\text{mod},0} = -\frac{\Omega_{\text{m},0}^2}{\Omega_{\text{loops},0}}$). One should note that in the case considered Λ also enters in non-standard fashion. It means that in this case we do not have the simple reduction to one model with different interpretations of Ω_{mod} , and the same Friedmann equation. Then, we are no longer comparing simple $(1+z)^6$ modifications, but rather different curved models. This case (under the prior $\Omega_{k,0} > 0$) are analysed and presented separately.

When one wants to compare the results obtained for different models, an interesting question is: how significant is the improvement of the fit due to the new model? To answer this question one can use the information criteria. The most popular information criteria used in everyday statistical practise are the Akaike information criterion (AIC) [61] and the Bayesian information criterion (BIC) [62]. These criteria can be used also for selection of model parameters by providing the preferred data fit.

Usually, incorporating new parameters increases the quality of the fit. The question is, if it increases it significantly enough. The information criteria put a threshold which must be exceeded in order to assert an additional parameter to be important in explanation of a phenomenon. The discussion of how high this threshold should be caused the appearing of many different criteria. For us, it suffices to check whether the AIC and BIC provide sufficient arguments for incorporation of the new parameters. The power of using information criteria of model selection was demonstrated by Liddle [63] and Parkinson et al. [64]. Please note that in any case some future observational data may give arguments in favour of additional parameters. Also, theoretical considerations could lead to inclusion of new parameters, even it is not necessary from the point of view of the present observations.

The AIC is defined in the following way [61]

$$\text{AIC} = -2 \ln \mathcal{L} + 2d, \quad (16)$$

where \mathcal{L} is the maximum likelihood and d is a number of free model parameters. The best model with a parameter set providing the preferred fit to the data is the one that minimizes the AIC. It is interesting that the AIC also arises from an approximate minimization of the Kullback-Leibler information entropy [65].

The BIC introduced by Schwarz [62] is defined as

$$\text{BIC} = -2 \ln \mathcal{L} + d \ln N, \quad (17)$$

where N is the number of data points used in the fit. Comparing these criteria, one should note that the AIC tends to favour models with a large number of parameters unlike the BIC, which penalizes new parameters more strongly. It is the reason why the BIC provides a more useful approximation to the full statistical analysis in the case of no priors on the set of model parameters [64]. This means that while the AIC is useful in obtaining upper limit to the number of parameters which should be incorporated to the model, the BIC is more conclusive. One should note that only the relative value between the BIC of different models has statistical significance. The difference of 2 is

treated as a positive evidence (and 6 as a strong evidence) against the model with the larger value of the BIC [66, 67]. If we do not find any positive evidence from information criteria the models are treated as identical and eventually additional parameters are treated as not significant. The using of the BIC seems to be especially suitable whenever the complexity of reference does not increase with the size of data set. The problem of classification of the cosmological models on the light of information criteria on the base of the astronomical data was discussed in our previous papers [68, 69, 70, 71, 72, 73].

Our results are presented in Table V. Please note that the results of statistical analysis for LQG models with non-zero curvature (under prior $\Omega_{k,0} > 0$) are presented as a separate case. It is clear that in the light of information criteria, the $(1+z)^6$ term does not increase the fit significantly. It confirms, what could be expected, that this term, if it exist, is small in the present epoch.

In the Bayesian framework, the quality of models can be compared with help of evidence [66, 74]. We can define the a posteriori odds for two models – M_i and M_j – the so called Bayes factor B_{ij} [75]. If a priori we do not favor any model it reduces to the evidence ratio. Schwarz [62] showed that for observations coming from a linear exponential distribution family in the asymptotic approximation $N \rightarrow \infty$ the logarithm of evidence is given by

$$\ln E = \ln \mathcal{L} - \frac{d}{2} \ln N + O(1). \quad (18)$$

It is easy to show that in this case we have a simple relation between the Bayes factor and the BIC

$$2 \ln B_{ij} = -(\text{BIC}_i - \text{BIC}_j) \quad (19)$$

If B_{ij} is greater than 3 it is considered a positive evidence in favor of M_i model, while $B_{ij} > 20$ gives strong, and $B_{ij} > 150$ very strong evidence in favor of model M_i [72]. We present our results in Table VI. In all cases we obtain positive evidence in favour of the Λ CDM model over the bouncing cosmology with the term ρ^2 . Only in the case when additional parameter of the theory (responsible for the term $(1+z)^6$) is fixed the obtained model is indistinguishable from the Λ CDM model. It supports the results obtained with help of the AIC and BIC. This result is valid both with and without the prior of $\Omega_{\text{mod},0} \leq 0$. One should note that because of a non-Gaussian distribution of the a posteriori PDF function for $\Omega_{\text{mod},0}$ the results obtain with help of Bayes factor (especially with the priors $\Omega_{\text{mod},0} \leq 0$) should be treated with caution as only additional support for results obtain with the AIC and BIC.

Please also note that if $\Omega_{\text{mod},0} < 0$, then we obtain a bouncing scenario [76, 77, 78] instead of a big bang. For $\Omega_{\text{m},0} = 0.3$, $\Omega_{\text{mod},0} = -0.26 \times 10^{-9}$ and $h = 0.65$ bounces ($H^2 = 0$) appear for $z \simeq 260$. In this case, the BBN epoch never occurs and all BBN predictions would be lost.

III. LOCATION OF CMB PEAKS AND BBN IN THE MAG MODEL

The results obtain in the previous section lead to the conclusion that we should obtain stronger constrains for model parameter especially for $\Omega_{\text{mod},0}$. One should note that for doing so, it is useful to analyse the location of the first peak in the CMB power spectrum and the predictions of the BBN. Stronger constraints for model parameters in the MAG model was obtained in Ref. [44].

The idea of testing models using the location of the first peak in the CMB power spectrum is based on the fact that the hotter and colder spots in the CMB can be interpreted as acoustic oscillation in the primeval plasma during the last scattering. Peaks in the power spectrum correspond to maximum density of the wave. In the Legendre multipole space these peaks correspond to the angle subtended by the sound horizon at the last scattering. Further peaks correspond to higher harmonics of the principal oscillations. The locations of these peaks are very sensitive to the variations in the model parameters. Therefore, the position of the first peak can be used as another way to constrain cosmological models.

In the MAG model, assuming $\Omega_{\text{m},0} = 0.3$ and $h = 0.72$ for the standard ΛCDM universe, the correct positions of the first peak was obtained in [44] as $\ell_1 = 220$. From the SNIa data analysis, it was found that the Hubble constant has lower value. Assuming that $H_0 = 65 \text{ km/s Mpc}$ (or $h = 0.65$) and consider the standard ΛCDM model, with $\Omega_{\text{m},0} = 0.3$, one gets $\ell_1 = 225$ [44].

Some discrepancy between the observational and theoretical results was found in this case. Now it was interesting to check whether the presence of the fictitious fluid $\Omega_{\psi,0}$ changes the locations of the peaks. If we choose the $H_0 = 65 \text{ km/s Mpc}$ then agreement with the observation of the location of the first peak could be obtained for three non-zero values of the parameter $\Omega_{\psi,0}$. Two positive and one negative values of this parameter for which the MAG model is admissible are 3×10^{-11} , 7×10^{-14} and -1.4×10^{-10} .

One should note that agreement with prediction of big-bang nucleosynthesis (BBN) is also crucial for testing of the model. Of course the big-bang nucleosynthesis is a very well tested area of cosmology and does not allow for any significant deviation from the standard expansion law apart from very early times (i.e., before the onset of BBN). The predictions of standard BBN are in good agreement with observations of the abundance of light elements. Therefore, all nonstandard terms added to the Friedmann equation should give only negligible small modifications during the BBN epoch to leave the nucleosynthesis process unchanged.

In our opinion the consistency with BBN is a crucial issue in the models where the nonstandard term a^{-6} is added in the Friedmann equation. It is clear that such a term has either accelerated

$(\Omega_{\text{mod},0} > 0)$ or decelerated ($\Omega_{\text{mod},0} < 0$) impact on the Universe expansion. Going backwards in time this term would become dominant at some redshift. If it had happened before the BBN epoch, the radiation domination would have never occurred and all the BBN predictions would be lost.

If we assume that the BBN result are preserved in our model, we obtain another constraint on the amount of $\Omega_{\psi,0}$. Let us assume that the model modification is negligibly small during the BBN epoch and the nucleosynthesis process is unchanged. It means that the contribution of the term Ω_{mod} cannot dominate over the radiation term $\Omega_{r,0} \approx 10^{-4}$ before the beginning of BBN ($z \simeq 10^8$)

$$\Omega_{\text{mod},0}(1+z)^6 < \Omega_{r,0}(1+z)^4 \quad \Rightarrow \quad |\Omega_{\psi,0}| < 10^{-20}.$$

The values of $\Omega_{\text{mod},0} \propto 10^{-3}$ obtained as best fits in the SNIa data analysis as well as the smallest nonzero value of $\Omega_{\psi,0} = 7 \times 10^{-14}$ calculated in the CMB analysis are unrealistic in the light of the above result. If we take into consideration the maximum likelihood analysis of SNIa data we have the possibility that the value of $\Omega_{\text{mod},0}$ is lower than $|10^{-20}|$ in the 2σ confidence interval.

One could argue that bounds from cosmography and CMB are weaker than those from nucleosynthesis. However, they are model independent. Of course, nucleosynthesis is a well tested area in cosmology [79] but it is described rather in the terms of standard physics without loops corrections. The first step toward the description of Big-Bang nucleosynthesis are given in the paper by Bojowald et al. [80]. The authors demonstrated that several correction to the equation of state parameter can arise from classical and quantum physics, for example loops quantum gravity allows one to compute quantum gravity corrections for Maxwell and Dirac field. So we cannot a priori assumed that the corresponding gravity corrections are negligible during the Big-Bang nucleosynthesis. In the present authors' opinion, the significance of bounds obtained from cosmography and CMB is that they are independent from the nucleosynthesis bounds and are related to different cosmological epochs.

IV. CONCLUSION

In the paper we have studied observational constraints on the FRW models with ρ^2 modifications, and obtained stronger limits on the magnitude of the term Ω_{mod} (scaling like $(1+z)^6$). We pointed out that astronomical observations allow us to test the total contributions of a fluid scaling like $(1+z)^6$ but we cannot separate particular terms of a negative sign. It is a simple consequence

TABLE I: Results of the statistical analysis of our model with $(1+z)^6$ like term obtained from χ^2 best fit. The upper section of the table represents the constraint $\Omega_{k,0} = 0$ (flat model).

sample	$\Omega_{k,0}$	$\Omega_{m,0}$	$\Omega_{\text{mod},0}$	$\Omega_{\Lambda,0}$	χ^2
SN	-	0.23	0.020	0.757	295.7
SN+RG	-	0.26	0.013	0.727	319.5
SN+RG+SDSS	-	0.28	0.009	0.711	319.6
SN+RG+SDSS+CMBR	-	0.30	0.4×10^{-8}	0.700	322.4
SN	0.34	0.00	0.044	0.646	295.6
SN+RG	0.29	0.05	0.034	0.626	319.5
SN+RG+SDSS	-0.03	0.28	0.011	0.739	319.5
SN+RG+SDSS+CMBR	0.04	0.30×10^{-6}	0.660	321.4	

TABLE II: Results of the statistical analysis of our model with $(1+z)^6$ term. The values of the model parameters are obtained from marginalized likelihood analysis. We present maximum likelihood value with 68.3% confidence ranges. The upper section of the table represents the constraint $\Omega_{k,0} = 0$ (flat model).

sample	$\Omega_{k,0}$	$\Omega_{m,0}$	$\Omega_{\text{mod},0}$	$\Omega_{\Lambda,0}$
SN	-	$0.22^{+0.07}_{-0.08}$	$0.021^{+0.020}_{-0.016}$	$0.75^{+0.06}_{-0.05}$
SN+RG	-	$0.27^{+0.05}_{-0.09}$	$0.013^{+0.018}_{-0.014}$	$0.73^{+0.05}_{-0.05}$
SN+RG+SDSS	-	$0.28^{+0.02}_{-0.02}$	$0.009^{+0.006}_{-0.006}$	$0.71^{+0.02}_{-0.02}$
SN+RG+SDSS+CMBR	-	$0.30^{+0.02}_{-0.01}$	$(0.279 \times 10^{-8})^{0.15 \times 10^{-5}}_{-3.01 \times 10^{-9}}$	$0.70^{+0.01}_{-0.02}$
SN	$0.10^{+0.19}_{-0.40}$	$0.00^{+0.34}_{-0.00}$	$0.024^{+0.016}_{-0.027}$	$0.77^{+0.14}_{-0.13}$
SN+RG	$0.14^{+0.18}_{-0.39}$	$0.00^{+0.35}_{-0.00}$	$0.019^{+0.014}_{-0.026}$	$0.73^{+0.14}_{-0.13}$
SN+RG+SDSS	$-0.03^{+0.05}_{-0.10}$	$0.28^{+0.02}_{-0.02}$	$0.011^{+0.011}_{-0.009}$	$0.74^{+0.02}_{-0.02}$
SN+RG+SDSS+CMBR	$0.01^{+0.04}_{-0.04}$	$0.30^{+0.02}_{-0.01}$	$(0.36 \times 10^{-6})^{+10.64 \times 10^{-6}}_{-3.6 \times 10^{-6}}$	$0.69^{+0.04}_{-0.03}$

of the fact that cosmography measures only the “average” density of matter. However, we showed that some stringent bounds on the value of this total contribution can be given.

There are several interpretations of the presence of the $(1+z)^6$ term in the Friedmann equation: 1) the brane theory; 2) the non-Riemannian theory of gravity; and 3) the loop quantum cosmology. All models give rise to a $(1+z)^6$ correction which effects are important in the very early universe and become unimportant in later evolution. The loop quantum cosmology is more fundamental theory than the classical theory. We are looking for the quantum cosmology predictions which are a priori in agreement with the classical picture of the present Universe (we showed the quantum loop effect are negligible at the present epoch).

Note that if we consider the phantom ρ^2 modification, it is also important in the future evolution

of the universe (Big-Rip singularities) [81].

We used Bayesian methods of model selection to answer the question: which cosmological model – with initial singularity or with bounce – is promoted by observational data? We have shown that models with a singularity are a “more economical” choice for the Universe but bouncing cosmology cannot be ruled out even on the 1σ level. However, because the the posteriori probability function for $\Omega_{\text{mod},0}$ is strongly non-Gaussian, probability of bounce, i.e. $P(\Omega_{\text{mod},0} < 0)$, is less than 1%. Our observational analysis clearly reflects the important role of independent observational data which enables us to refine the analysis of model parameters.

The analysis of SNIa data as well as both SNIa and FRIIb radio galaxies (with and without priors coming from baryon oscillation peaks and CMBR “shift parameter”) shows that the values of χ^2 statistics are lower for model with $(1+z)^6$ like term, than for the Λ CDM model. On the other hand, information criteria show that including such a term does not increase the quality of the fit significantly; or, alternatively, that the quality of the available data is not good enough for fitting this new term. BIC even favours the Λ CDM model over our model of bounce, although this preference is weak. These results lie in agreement with the fact that $(1+z)^6$ term is not significant in the present epoch of the Universe. Moreover, if the additional parameter of the theory $\Omega_{\text{mod},0}$ is fixed, like for the LQG model, than we obtained model is indistinguishable from the Λ CDM model in the Bayesian framework.

The combined analysis of SNIa data and FRIIb radio galaxies using baryon oscillation peaks and CMBR “shift parameter” gives rise to a concordance universe model which is almost flat with $\Omega_{\text{m},0} \simeq 0.3$. From the above mentioned combined analysis, we obtain the following constraint for the term which scales like $(1+z)^6$: $\Omega_{\text{mod},0} \in (-0.26 \times 10^{-9}; 0.31 \times 10^{-4})$

We confirm Bojowald’s assertion that effects of quantum gravity can be potentially tested, but the required bounds lie beyond the possibilities of high precision cosmology. What we obtain from cosmography is very weak and other tests may be useful. We have shown that the analysed scenario is compatible with the most recent low redshift observations of SN Ia, which are independent of physical processes in the early universe. We find the other limits on the value of $\Omega_{\psi,0}$ from measurements of CMB anisotropies and BBN. We obtain the strongest limits in this case, namely $\Omega_{\psi,0} \leq 10^{-20}$ from BBN. Of course BBN as well as CMB are a very well tested areas of cosmology which do not allow for significant and substantial changes. Still, one must remember that, although consistency with BBN and CMB is a crucial issue, in such an approach we a priori assume that brane models (for example) do not change the physics of the pre-recombination epochs.

The analysis was performed for flat and non-flat models with the tacit assumption that the

TABLE III: Results of the statistical analysis of the bouncing model with the negative $(1+z)^6$ term obtained from χ^2 best fit. The upper section of the table represents the constraint $\Omega_{k,0} = 0$ (flat model).

Sample	$\Omega_{k,0}$	$\Omega_{m,0}$	$\Omega_{\text{mod},0}$	$\Omega_{\Lambda,0}$	χ^2
SN	-	0.31	0.000	0.69	297.5
SN+RG	-	0.32	0.000	0.68	320.4
SN+RG+SDSS	-	0.30	0.000	0.70	322.4
SN+RG+SDSS+CMBR	-	0.30	0.000	0.70	322.5
SN	-0.28	0.43	0.000	0.85	296.0
SN+RG	-0.18	0.39	0.000	0.80	319.5
SN+RG+SDSS	0.06	0.29	0.000	0.65	321.1
SN+RG+SDSS+CMBR	0.00	0.30	0.000	0.70	322.5

curvature does not change the physics in any other way than introducing an appropriate term in the Friedmann equation. Since this is not the case for LQG, only the flat case considerations give bounds for the parameter Ω_{loops} . Although curved LQG models have been introduced [13, 50], they have significantly different matter components, making it impossible to compare them in our scheme.

Our general conclusion is that while cosmography can be generally used to test ρ^2 type contribution, such correction turns out to be very small. Moreover, because we use $H(z)$ function which probes only the average density, it is not possible to separate effects of LQG from other effects scaling like $(1+z)^6$, for example effects of the brane. One should note that in LQG theory $\Omega_{\text{loops},0}$ is fixed and gives an odd contribution to $\Omega_{\text{mod},0} \simeq 1.7 \times 10^{-124}$ only, which is far below the possibility to test by present cosmography. It means that any positive evidence for non zero $(1+z)^6$ term eventually obtained from cosmography can not be connected with LQG.

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TABLE IV: Results of the statistical analysis of the bouncing model with the negative $(1+z)^6$ term. The values of the model parameters are obtained from the marginalized likelihood analysis. We present the maximum likelihood values with 68.3% confidence ranges. The upper section of the table represents the constraint $\Omega_{k,0} = 0$ (flat model).

Sample	$\Omega_{k,0}$	$\Omega_{m,0}$	$\Omega_{\text{mod},0}$	$\Omega_{\Lambda,0}$
SN	-	$0.33^{+0.02}_{-0.02}$	$0.000_{-0.006}$	$0.67^{+0.02}_{-0.02}$
SN+RG	-	$0.33^{+0.03}_{-0.02}$	$0.000_{-0.006}$	$0.67^{+0.02}_{-0.03}$
SN+RG+SDSS	-	$0.31^{+0.02}_{-0.02}$	$0.000_{-0.002}$	$0.69^{+0.02}_{-0.02}$
SN+RG+SDSS+CMBR	-	$0.30^{+0.02}_{-0.01}$	$0.00000_{-0.22 \times 10^{-9}}$	$0.70^{+0.01}_{-0.02}$
SN	$-0.42^{+0.24}_{-0.22}$	$0.52^{+0.09}_{-0.09}$	$0.000_{-0.011}$	$0.91^{+0.11}_{-0.15}$
SN+RG	$-0.36^{+0.25}_{-0.20}$	$0.49^{+0.10}_{-0.11}$	$0.000_{-0.011}$	$0.85^{+0.12}_{-0.13}$
SN+RG+SDSS	$0.08^{+0.06}_{-0.05}$	$0.29^{+0.02}_{-0.02}$	$0.000_{-0.004}$	$0.63^{+0.05}_{-0.06}$
SN+RG+SDSS+CMBR	$0.00^{+0.02}_{-0.03}$	$0.30^{+0.02}_{-0.01}$	$0.00000_{-0.21 \times 10^{-9}}$	$0.70^{+0.02}_{-0.02}$

TABLE V: The values of AIC and BIC for the Λ CDM model and Bouncing Cosmology model (with the term $(1+z)^6$) without and with priors $\Omega_{\text{mod},0} \leq 0$. Separately the LQG model with fixed $\Omega_{\text{mod},0}$ ($\Omega_{k,0} > 0$) is considered. The upper section of the table represents the constraint $\Omega_{k,0} = 0$ (flat model).

	Λ CDM		Λ BCDM		Λ BCDM($\Omega_{\psi,0} \leq 0$)		LQG	
sample	AIC	BIC	AIC	BIC	AIC	BIC	AIC	BIC
SN	299.5	303.1	299.7	306.9	301.5	308.7	299.5	303.1
SN+RG	322.4	326.1	323.5	330.9	324.4	331.8	322.4	326.1
SN+RG+SDSS	324.4	328.1	323.6	332.0	326.4	333.8	324.4	328.1
SN+RG+SDSS+CMBR	324.5	328.2	326.4	333.8	326.5	333.9	324.5	328.2
SN	300.0	307.2	301.6	312.4	302.0	312.8	301.5	308.7
SN+RG	323.5	330.9	325.5	336.5	325.5	336.5	324.4	331.8
SN+RG+SDSS	325.1	332.5	325.5	336.5	327.1	338.1	325.1	332.5
SN+RG+SDSS+CMBR	326.5	333.9	327.4	338.4	328.5	339.5	326.5	333.9

Appendix. Basics of Loop Quantum Cosmology.

In this appendix we give some selected information about Loop Quantum Gravity (LQG) connected to the subject of this work. Our main challenge is to show how to obtain equation (1) considered in this paper (see also a nice summary [82]). We concentrate rather on the main steps and for detailed calculations we direct to the references.

Loop Quantum Gravity describe the gravitational field as a $SU(2)$ non-Abelian gauge field using

TABLE VI: The values of Bayes factor for models: 1) Λ CDM model, 2) Bouncing Cosmology model (with the term $(1+z)^6$) and 3) Bouncing Cosmology model (with $(1+z)^6$ term) with priors $\Omega_{\text{mod},0} \leq 0$, 4) LQG model with fixed $\Omega_{\text{mod},0}$ ($\Omega_{k,0} > 0$). The upper section of the table represents the constraint $\Omega_{k,0} = 0$ (flat model).

sample	B_{12}	B_{13}	B_{14}	B_{23}	B_{42}	B_{43}
SN	6.69	16.44	1.00	2.46	6.69	16.44
SN+RG	11.02	17.29	1.00	1.57	11.02	17.29
SN+RG+SDSS	7.03	17.29	1.00	2.46	7.03	17.29
SN+RG+SDSS+CMBR	16.44	17.29	1.00	1.05	16.44	17.29
SN	13.46	16.44	2.12	1.22	6.36	7.77
SN+RG	16.44	17.29	1.57	1.00	10.49	11.02
SN+RG+SDSS	7.39	17.29	1.00	2.23	7.39	17.29
SN+RG+SDSS+CMBR	9.49	17.29	1.00	1.73	9.49	17.29

background independent methods. The canonical fields are so called Ashtekar variables (A, E) [83] which take value in $\mathfrak{su}(2)$ and $\mathfrak{su}(2)^*$ algebras respectively. These variables are analogues of the four potential and electric field in electrodynamics. The Ashtekar variables are strictly connected with triad representation. However in LQG gauge fields describe only spatial part Σ when time is treated separately. To quantise this theory in the background independent way ones introduce holonomies of connection A

$$h_\alpha[A] = \mathcal{P} \exp \int_\alpha A \quad \text{where 1-form } A = \tau_i A_a^i dx^a \quad (20)$$

and conjugated fluxes

$$F_S^i[E] = \int_S dF^i \quad \text{where 2-form } dF_i = \epsilon_{abc} E_i^a dx^b \wedge dx^c. \quad (21)$$

which are background independent observables. In equation (20) we have introduced $\tau_i = -\frac{i}{2}\sigma_i$ where σ_i are Pauli matrices. The quantisation of theory lead to important result that volumes and areas have discrete spectrum. For further calculations it will be important to notice that there exists a minimal area $\Delta = 2\sqrt{3}\pi\gamma l_{\text{Pl}}^2$ [84]. For introductory review on loop quantisation see [85].

Dynamics of theory is contained in the scalar constraint

$$H_G = \frac{1}{16\pi G} \int_\Sigma d^3x N(x) \frac{E_i^a E_j^b}{\sqrt{|\det E|}} \left[\varepsilon^{ij}_k F_{ab}^k - 2(1+\gamma^2) K_{[a}^i K_{b]}^j \right] \quad (22)$$

where F is a field strength $F = dA + \frac{1}{2}[A, A]$ and K is an extrinsic curvature. Constant γ in equation (22) is so called Barbero-Immirzi parameter, $\gamma = \ln 2/(\pi\sqrt{3})$.

We want to show now how to apply LQG to case of the flat FRW model considered in this paper. The results presented below base on papers [7, 8, 9] where reader can find detailed calculations and analysis. The FRW $k = 0$ spacetime metric can be written as

$$ds^2 = -N^2(x)dt^2 + q_{ab}dx^adx^b \quad (23)$$

where $N(x)$ is the lapse function (here we choose gauge $N(x) = 1$) and the spatial part of the metric is expressed as

$$q_{ab} = \delta_{ij}\omega_a^i\omega_a^j = a^2(t)^oq_{ab}\delta_{ij}^o\omega_a^i\omega_a^j. \quad (24)$$

In this expression ${}^oq_{ab}$ is fiducial metric and ${}^o\omega_a^i$ are co-triads dual to the triads ${}^oe_i^a$, ${}^o\omega^i({}^oe_j) = \delta_j^i$ where ${}^o\omega^i = {}^o\omega_a^i dx^a$ and ${}^oe_i = {}^oe_i^a \partial_a$. In the case considered the Ashtekar variables are

$$A \equiv \Gamma + \gamma K = cV_0^{-1/3} {}^o\omega_a^i \tau_i dx^a, \quad (25)$$

$$E \equiv \sqrt{|\det q|}e = pV_0^{-2/3} \sqrt{{}^oq} {}^oe_i^a \tau_i \partial_a \quad (26)$$

where V_0 is volume of fiducial cell and Γ is spin connection. The parameters (c, p) can be since now considered as canonical variables with fundamental Poisson bracket $\{c, p\} = 8\pi G\gamma/3$. In quantum theory p is replaced by operator \hat{p} which acting on the eigenvector $|\mu\rangle$ gives

$$\hat{p}|\mu\rangle = \mu \frac{8\pi l_{\text{Pl}}^2 \gamma}{6} |\mu\rangle. \quad (27)$$

where $\mu \in \mathbb{R}$ and eigenvectors fulfils relation of orthogonality $\langle \mu_i | \mu_j \rangle = \delta_{\mu_i, \mu_j}$. There is no well defined operator of variable c , instead of this there exists another fundamental operator defined as

$$\widehat{\exp \frac{i\lambda c}{2}} |\mu\rangle = |\mu + \lambda\rangle. \quad (28)$$

With use of definition (20) we can calculate holonomy for connection (25) in particular direction ${}^oe_i^a \partial_a$

$$h_i^{(\lambda)} = \mathbb{I} \cos\left(\frac{\lambda c}{2}\right) + \tau_i \sin\left(\frac{\lambda c}{2}\right). \quad (29)$$

Holonomy is well defined operator which acting on vector $|\mu\rangle$ gives

$$\hat{h}_i^{(\lambda)} |\mu\rangle = \frac{1}{2} (|\mu + \lambda\rangle + |\mu - \lambda\rangle) + \frac{1}{i} (|\mu + \lambda\rangle - |\mu - \lambda\rangle) \tau_i \quad (30)$$

where we have used definition (28). From particular holonomies (29) we can construct holonomy along the closed curve $\alpha = \square_{ij}$. This holonomy can be written as

$$h_{\square_{ij}}^{(\mu)} = h_i^{(\mu)} h_j^{(\mu)} h_i^{(\mu)-1} h_j^{(\mu)-1} = \mathbb{I} + \mu^2 V_0^{2/3} F_{ab}^k \tau_k {}^oe_i^{ao} {}^oe_j^b + \mathcal{O}(\mu^3) \quad (31)$$

and inverting this equation we obtain expression for field strength

$$F_{ab}^k \approx -2 \frac{\text{tr} \left[\tau_k \left(h_{\square_{ij}}^{(\mu)} - \mathbb{I} \right) \right]_o \omega_a^i \omega_b^j}{\mu^2 V_0^{2/3}} = \frac{\sin^2(\mu c)}{\mu^2 V_0^{2/3}} \epsilon_{kij}^o \omega_a^i \omega_b^j \quad (32)$$

In the limit $\mu \rightarrow 0$ we retrieve classical formula. However this limit does not exist in quantum theory because of area gap Δ . LQG tell us that we should stop shrinking the loop $\mu \rightarrow \bar{\mu}$ when physical area $p\bar{\mu}^2 = \Delta$. Then inserting expressions for field strength (32) and for A and K to (22) we obtain the effective Hamiltonian with holonomy corrections

$$H_{\text{eff}} = -\frac{3}{8\pi G \gamma^2} \sqrt{p} \left[\frac{\sin(\bar{\mu}c)}{\bar{\mu}} \right]^2 + p^{3/2} \rho_m \quad (33)$$

where we have also added matter part. Additional feature of theory is Hamiltonian constraint $H_{\text{eff}} = 0$ which implies

$$\frac{1}{\gamma^2 p} \left[\frac{\sin(\bar{\mu}c)}{\bar{\mu}} \right]^2 = \frac{8\pi G}{3} \rho_m. \quad (34)$$

Now we can use the Hamilton equation to determinate dynamics of the canonical variable p

$$\dot{p} = \{p, H_{\text{eff}}\} = \frac{2}{\gamma} \frac{\sqrt{p}}{\bar{\mu}} \sin(\bar{\mu}c) \cos(\bar{\mu}c). \quad (35)$$

Combining equations (34) and (35) we finally obtain the modified Friedmann equation with holonomy correction

$$H^2 = \frac{8\pi G}{3} \rho_m \left(1 - \frac{\rho_m}{\rho_c} \right) \quad (36)$$

where we have introduced critical energy density

$$\rho_c = \frac{3}{8\pi G \gamma^2 \bar{\mu}^2 p} = \frac{3}{8\pi G \gamma^2 \Delta} = \frac{\sqrt{3}}{16\pi^2 \gamma^3 l_{\text{Pl}}^4}. \quad (37)$$

For further details of Loop Quantum Cosmology we recommend the introductory review [15].

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